

Key

3/1/2011

Analyzing Exponential Functions

Notes

Characteristics of graphs

Characteristic	How to write the answer
◦ Domain -	(#, #) All reals, $x < \#$, $x > \#$
◦ Range -	(#, #) All reals, $y < \#$, $y > \#$
◦ Zero's - x intercepts	(#, 0)
◦ Y intercept -	(0, #)
◦ Rate of change - (slope formula)	# or fraction
◦ Interval of inc/dec -	(#, #)
◦ End Behavior -	
◦ Asymptote -	$y = \#$

$y = 2^x$ Characteristics of Exponential Functions

Domain: \mathbb{R}

Range: $y > 0$

Asymptotes: $y = 0$ Horizontal Asymptote

Zeros: none

Intercepts: (0, 1)

$y = 2^x$ Characteristics of Exponential Functions

Intervals of Increase and Decrease
Increases from $-\infty$ to ∞

Rate of change over the interval $-2 \leq x \leq 2$
Use the points $(-2, \frac{1}{4})$ and $(2, 4)$ find the rate of change (aka...slope)

When $x = -2$ When $x = 2$
 $y = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} = .25$ $y = 2^2 = 4$

$m = \frac{4 - .25}{2 - (-2)} = \frac{3.75}{4} = .938$

$y = 2^x$ Characteristics of Exponential Functions

End Behaviors

$x \rightarrow \infty f(x) \rightarrow \infty$ approaches

As the value of x approaches positive infinity (∞) the value of the function approaches ∞ .

$x \rightarrow -\infty f(x) \rightarrow 0$

As the value of x approaches negative infinity ($-\infty$) the value of the function approaches but NEVER reaches 0.

$y = \left(\frac{1}{2}\right)^x$ Characteristics of Exponential Functions

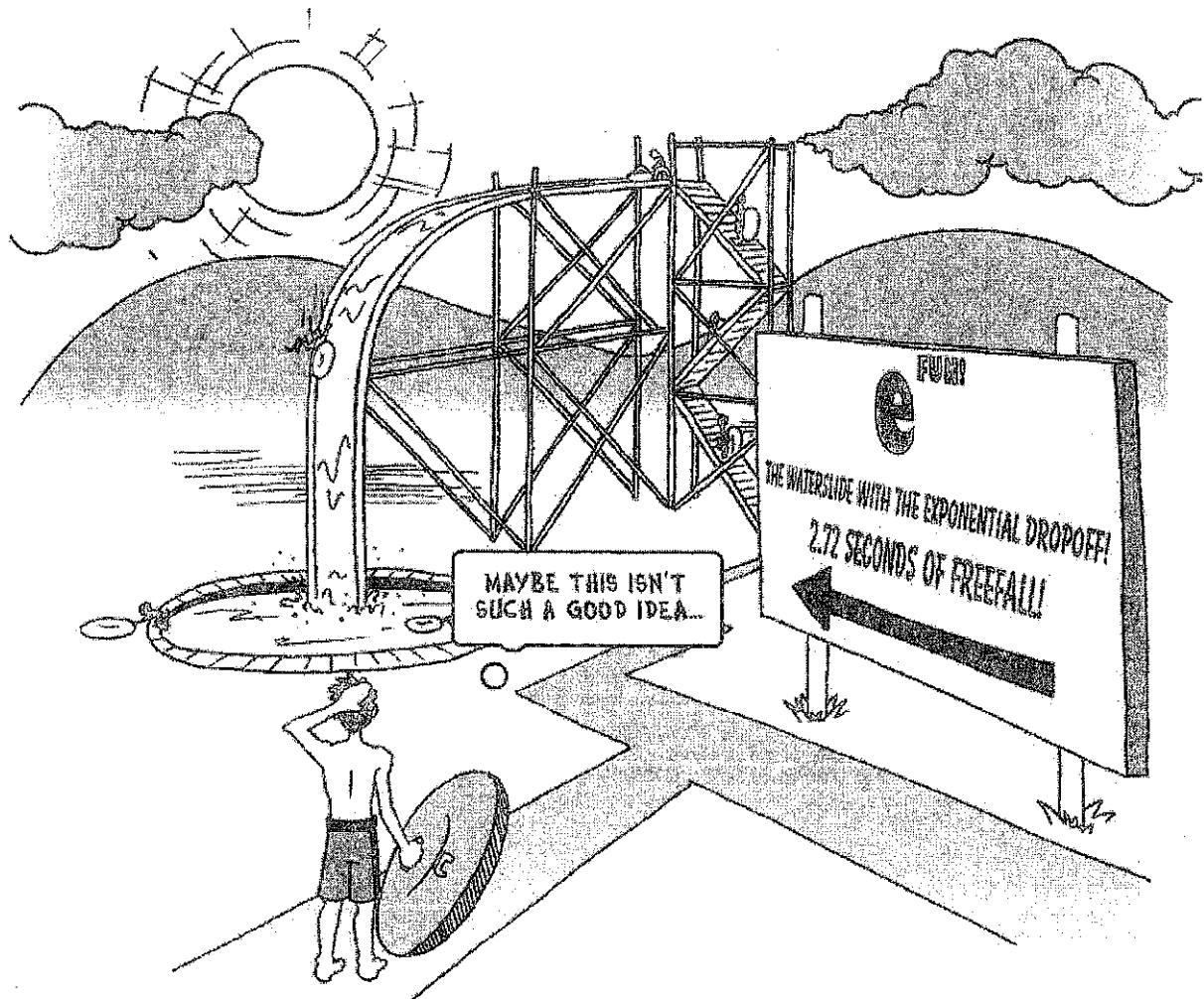
Domain: \mathbb{R}

Range: $y > 0$

Asymptotes: $y = 0$ Horizontal Asymptote

Zeros: none

Intercepts: (0, 1)



MAYBE THIS ISN'T
SUCH A GOOD IDEA...

FUHH
THE WATERSIDE WITH THE EXPONENTIAL DROPOFF!
2.72 SECONDS OF FREEFALL!

Characteristics of Exponential Functions

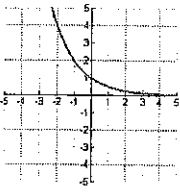
$y = \left(\frac{1}{2}\right)^x$

Intervals of Increase and Decrease
Decreases from $-\infty$ to ∞

Rate of change over the interval $-2 \leq x \leq 2$
Use the points $(-2, \frac{1}{4})$ and $(2, \frac{1}{4})$ find the rate of change (aka...slope)

When $x = -2$ When $x = 2$
 $y = \left(\frac{1}{2}\right)^{-2} = \frac{1}{2^{-2}} = \frac{2^2}{1} = 4 = 4$ $y = \left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$

$m = \frac{\frac{1}{4} - 4}{2 - (-2)} = \frac{-\frac{15}{4}}{4} = -\frac{15}{16}$



Characteristics of Exponential Functions

$y = \left(\frac{1}{2}\right)^x$

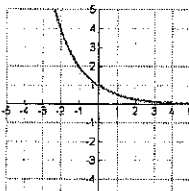
End Behaviors

$x \rightarrow \infty f(x) \rightarrow 0$ (approaches)

As the value of x approaches positive infinity (∞) the value of the function approaches but NEVER reaches 0.

$x \rightarrow -\infty f(x) \rightarrow \infty$

As the value of x approaches negative infinity ($-\infty$) the value of the function approaches ∞ .



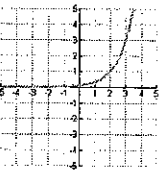
YOUR TURN! **Characteristics of Exponential Functions**

$y = 3^{x-2}$

Domain: All real numbers
Range: $y > 0$
Asymptotes: $y = 0$
Zeros: none
Intercepts: $3^0 = 3^{-2} = \frac{1}{9}$
Intervals of Increase and Decrease:
increase from $-\infty$ to ∞

Rate of change over the interval $-2 \leq x \leq 2$:
 $(-2, 1/81) (2, 1)$ $\frac{1 - \frac{1}{81}}{2 - (-2)} = \frac{\frac{80}{81}}{4} = \frac{20}{81}$

End Behaviors: $x \rightarrow \infty f(x) \rightarrow \infty$
 $x \rightarrow -\infty f(x) \rightarrow 0$



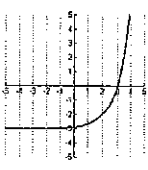
YOUR TURN! **Characteristics of Exponential Functions**

$y = 3^{x-2} - 3$

Domain: All real numbers
Range: $y > -3$
Asymptotes: $y = -3$
Zeros: (3,0)
Intercepts: $3^{x-2} - 3 = 3^{-2} - 3 = \frac{1}{9} - 3 = -\frac{26}{9}$
Intervals of Increase and Decrease:
increase from $-\infty$ to ∞

Rate of change over the interval $-2 \leq x \leq 2$:
 $(-2, -242/81) (2, -2)$ $\frac{-2 - \frac{-242}{81}}{2 - (-2)} = \frac{\frac{230}{81}}{4} = \frac{115}{162}$

End Behaviors: $x \rightarrow \infty f(x) \rightarrow \infty$
 $x \rightarrow -\infty f(x) \rightarrow -3$



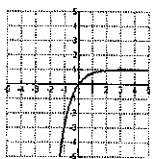
YOUR TURN! **Characteristics of Exponential Functions**

$y = -\left(\frac{1}{4}\right)^x + 1$

Domain: All real numbers
Range: $y < 1$
Asymptotes: $y = 1$
Zeros: (0,0)
Intercepts: $-\left(\frac{1}{4}\right)^0 + 1 = -1 + 1 = 0$ (0,0)
Intervals of Increase and Decrease:
increase from $-\infty$ to ∞

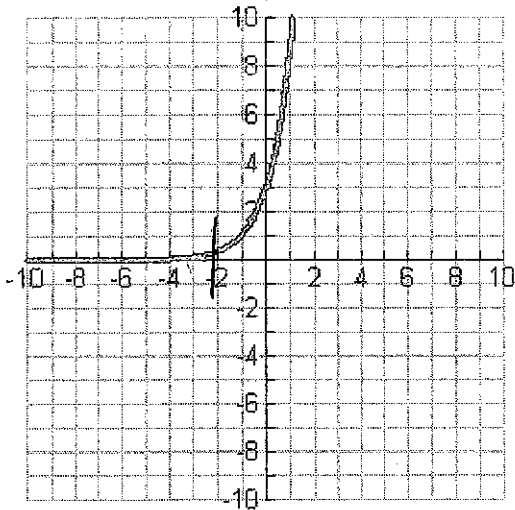
Rate of change over the interval $-2 \leq x \leq 2$:
 $(-2, 17) (2, 17/16)$ $\frac{17 - \frac{17}{16}}{2 - (-2)} = \frac{\frac{255}{16}}{4} = \frac{255}{64}$

End Behaviors: $x \rightarrow \infty f(x) \rightarrow 1$
 $x \rightarrow -\infty f(x) \rightarrow -\infty$



Analyzing Growth and Decay Exponential Functions

1. $y = 3^{x+1}$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptotes: $y = 0$

Zeros: \emptyset

y-Intercepts: $(0, 3)$

Intervals of increase and decrease $\uparrow (-\infty, \infty)$

~~Rate of change of the interval $-2 \leq x \leq 2$~~

End behavior

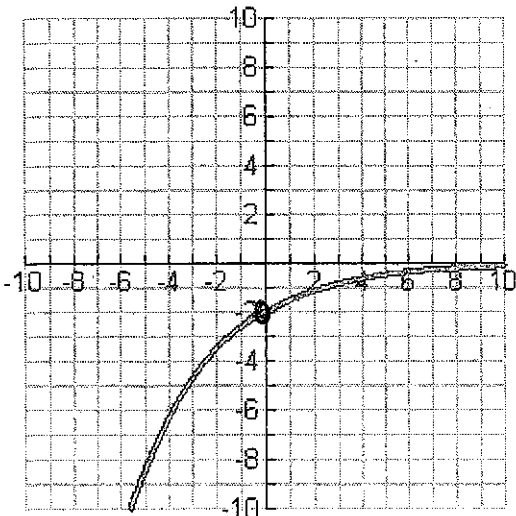
As $x \rightarrow -\infty$ $y \rightarrow 0$

As $x \rightarrow \infty$ $y \rightarrow \infty$

$$\begin{matrix} (-2, \frac{1}{3}) & (2, 27) \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

$$\frac{27 - \frac{1}{3}}{2 - -2} = \frac{26\frac{2}{3}}{4} =$$

2. $y = -2\left(\frac{3}{4}\right)^x$



Domain $(-\infty, \infty)$

Range: $(-\infty, 0)$

Asymptotes: $y = 0$

Zeros: \emptyset

y-Intercepts: $(0, -2)$

Intervals of increase and decrease $\uparrow (-\infty, \infty)$

~~Rate of change of the interval $-2 \leq x \leq 2$~~

End behavior

As $x \rightarrow \infty$

$y \rightarrow -\infty$

As $x \rightarrow -\infty$

$y \rightarrow 0$

$$\begin{matrix} (-2, -3\frac{5}{4}) & (2, -1\frac{1}{8}) \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

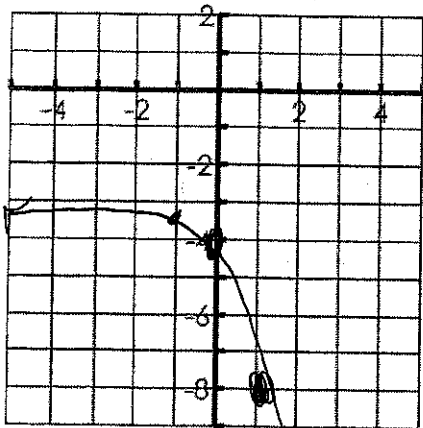
$$\frac{-1\frac{1}{8} - -3\frac{5}{4}}{2 - -2} =$$

Name: _____

Date: _____

Graphing and Characteristics of Exponential Equations

1. $y = -5^x - 3$



Domain $(-\infty, \infty)$ Range $(-\infty, -3)$

Asymptote $y = -3$

Increasing or Decreasing $(-\infty, \infty)$

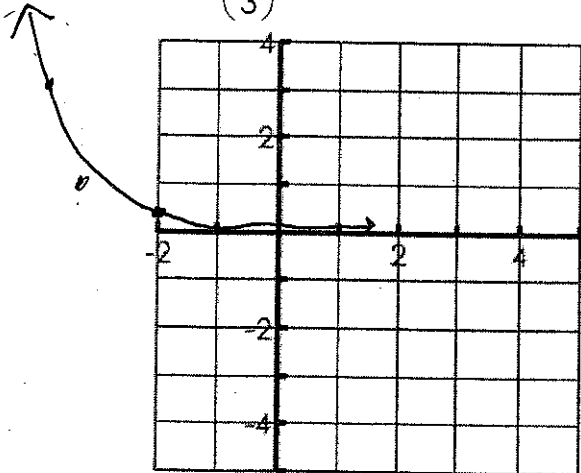
X-intercept \emptyset Y-intercept $(0, -4)$

End Behavior $x \rightarrow -\infty, f(x) \rightarrow -3$
 $x \rightarrow \infty, f(x) \rightarrow \infty$

Initial Amount: -1
 growth/decay factor 5 $-1(5)^x - 3$

x	y
-1	-3.2
0	-4
1	-8

2. $y = \left(\frac{1}{3}\right)^{x+3}$



Domain $(-\infty, \infty)$ Range $(0, \infty)$

Asymptote $y = 0$

Increasing or Decreasing $(-\infty, \infty)$

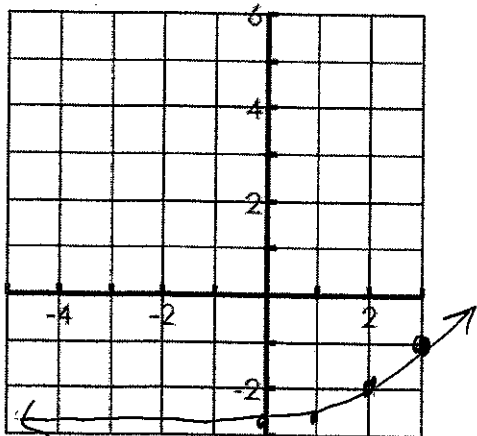
X-intercept \emptyset Y-intercept \emptyset

End Behavior $x \rightarrow -\infty, f(x) \rightarrow \infty$
 $x \rightarrow \infty, f(x) \rightarrow 0$

Initial Amount: 1
 growth/decay factor $\frac{1}{3}$ $y = 1\left(\frac{1}{3}\right)^{x+3}$

x	y
-4	3
-3	1
-2	0.33

3. $y = 4^{x-2} - 3$



Domain $(-\infty, \infty)$ Range $(-3, \infty)$

Asymptote $y = -3$

Increasing or Decreasing $(-\infty, \infty)$

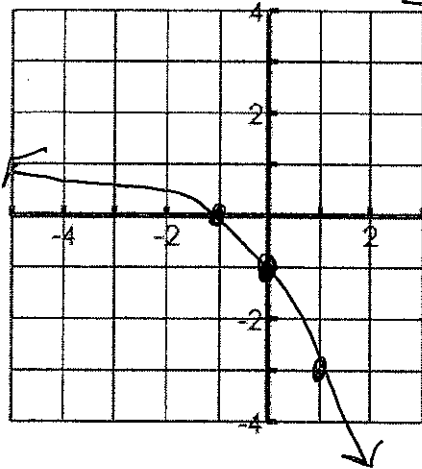
X-intercept \emptyset Y-intercept $(0, -2.9)$ End Behavior

$x \rightarrow -\infty, f(x) \rightarrow -3$
 $x \rightarrow \infty, f(x) \rightarrow \infty$

Initial Amount: 1
 growth/decay factor 4

x	y
0	-2.9
1	-2.75
2	-2
3	-1

4. $y = -2^{x+1} + 1$



x	y
-1	0
0	-1
1	-3

Domain $(-\infty, \infty)$ Range $(-\infty, 1)$

Asymptote $y = 1$

Increasing or Decreasing $(-\infty, \infty)$

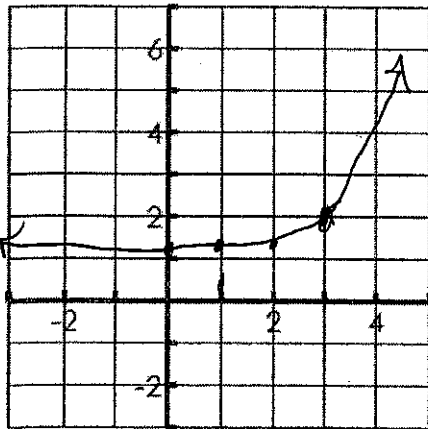
X-intercept $(-1, 0)$ Y-intercept $(0, -1)$

End Behavior $x \rightarrow -\infty, f(x) \rightarrow 1$
 $x \rightarrow \infty, f(x) \rightarrow -\infty$

Initial Amount: -1

(growth/decay factor 2)

5. $y = 3^{x-3} + 1$



x	y
0	1.04
1	1.3
2	1.9
3	2

Domain $(-\infty, \infty)$ Range $(1, \infty)$

Asymptote $y = 1$

Increasing or Decreasing $(-\infty, \infty)$

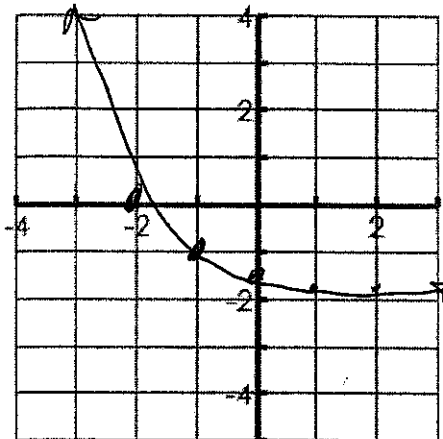
X-intercept \emptyset Y-intercept $(0, 1.04)$

End Behavior $x \rightarrow -\infty, f(x) \rightarrow 1$
 $x \rightarrow \infty, f(x) \rightarrow \infty$

Initial Amount: 1

growth/decay factor 3

6. $y = \left(\frac{1}{2}\right)^{x+1} - 2$



x	y
-2	0
-1	-1
0	-1.5
1	-1.75
2	-1.875

Domain $(-\infty, \infty)$ Range $(-2, \infty)$

Asymptote $y = -2$

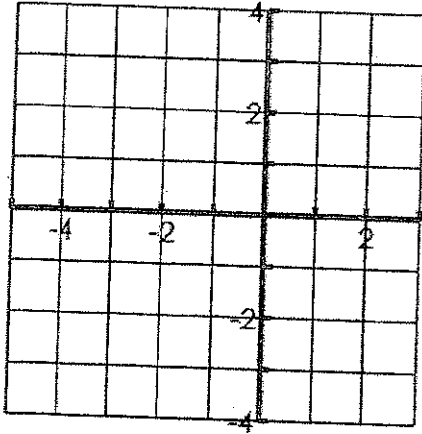
Increasing or Decreasing $(-\infty, \infty)$

X-intercept $(-2, 0)$ Y-intercept $(0, -1.5)$

End Behavior $x \rightarrow -\infty, f(x) \rightarrow \infty$
 $x \rightarrow \infty, f(x) \rightarrow -2$

Initial Amount: 1 growth/decay factor $\frac{1}{2}$

4. $y = -2^{x+1} + 1$



Domain _____ Range _____

Asymptote _____

Increasing or Decreasing _____

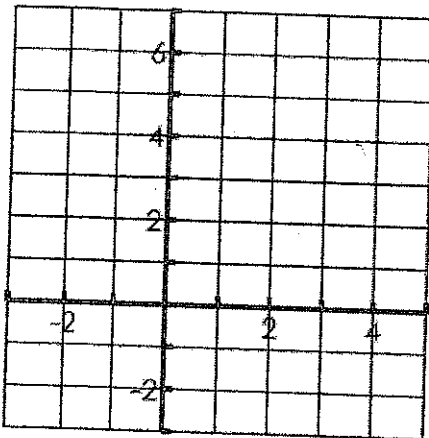
X-intercept _____ Y-intercept _____

End Behavior $x \rightarrow \text{_____}, f(x) \rightarrow \text{_____}$
 $x \rightarrow \text{_____}, f(x) \rightarrow \text{_____}$

Initial Amount: _____

growth/decay factor _____

5. $y = 3^{x-3} + 1$



Domain _____ Range _____

Asymptote _____

Increasing or Decreasing _____

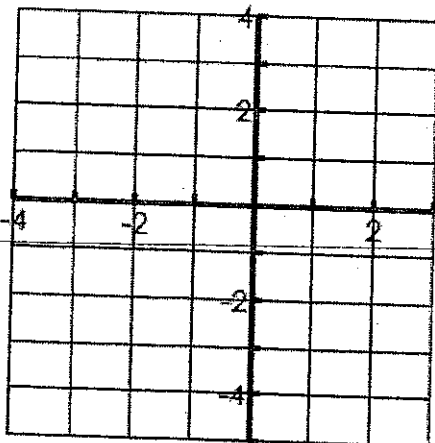
X-intercept _____ Y-intercept _____

End Behavior $x \rightarrow \text{_____}, f(x) \rightarrow \text{_____}$
 $x \rightarrow \text{_____}, f(x) \rightarrow \text{_____}$

Initial Amount: _____

growth/decay factor _____

6. $y = \left(\frac{1}{2}\right)^{x+1} - 2$



Domain _____ Range _____

Asymptote _____

Increasing or Decreasing _____

X-intercept _____ Y-intercept _____

End Behavior $x \rightarrow \text{_____}, f(x) \rightarrow \text{_____}$
 $x \rightarrow \text{_____}, f(x) \rightarrow \text{_____}$

Initial Amount: _____ growth/decay factor _____