

Solving Quadratic Inequalities in One Variable

Quadratic inequalities in one variable are inequalities which can be written in one of the following forms, where a , b , and c are real numbers:

$ax^2 + bx + c > 0$ $ax^2 + bx + c < 0$ $ax^2 + bx + c \geq 0$ $ax^2 + bx + c \leq 0$

To solve a quadratic inequality, follow these steps:
 "greater than" "less than" "greater than or equal to" "less than or equal to"

STEP 1 Replace the inequality symbol with an equal sign.

STEP 2 Move all terms to one side and set equal to 0.

STEP 3 Solve the quadratic equation using square roots, factoring, or the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

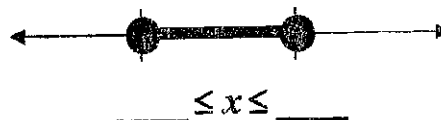
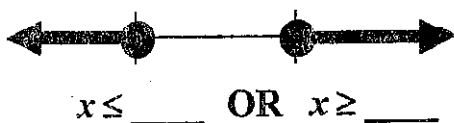
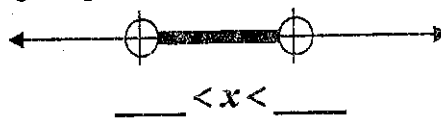
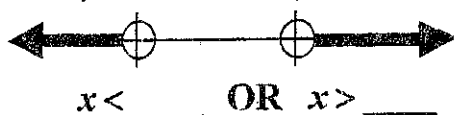
STEP 4 Draw a number line. Place the answers on the number line using **open circles** or **closed circles**.

Use open circles for the symbols $<$ or $>$.

Use closed circles for the symbols \leq or \geq .

STEP 5 Test an x -value in each section of the number line using the **inequality**.
 Shade the section(s) of the number line that make a true statement.

STEP 6 Write your answer using one of the following inequalities:



EXAMPLE 1

$x^2 > x + 2$ (open circle)

$x^2 = x + 2$
 $-x - 2 - x - 2$
 $x^2 - x - 2 = 0$
 $(x - 2)(x + 1) = 0$
 $x - 2 = 0$ $x + 1 = 0$
 $x = 2$ $x = -1$

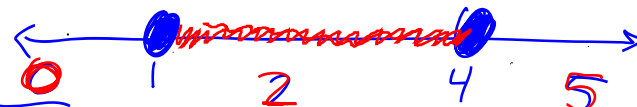


$x^2 > x + 2$ $(-2)^2 > (-2) + 2$ $4 > 0$ <u>True</u>	$x^2 > x + 2$ $0^2 > 0 + 2$ $0 > 2$ <u>False</u>	$x^2 > x + 2$ $3^2 > 3 + 2$ $9 > 5$ <u>True</u>
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EXAMPLE 2

$x^2 - 5x \leq -4$ (closed circle)

$x^2 - 5x = -4$
 $+4 +4$
 $x^2 - 5x + 4 = 0$
 $(x - 4)(x - 1) = 0$
 $x - 4 = 0$ $x - 1 = 0$
 $x = 4$ $x = 1$



$x^2 - 5x \leq -4$ $(0)^2 - 5(0) \leq -4$ $0 - 0 \leq -4$ $0 \leq -4$ <u>False</u>	$x^2 - 5x \leq -4$ $(2)^2 - 5(2) \leq -4$ $4 - 10 \leq -4$ $-6 \leq -4$ <u>True</u>	$x^2 - 5x \leq -4$ $5^2 - 5(5) \leq -4$ $25 - 25 \leq -4$ $0 \leq -4$ <u>False</u>
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EXAMPLE 3 $2x^2 + 13x < -6$

open circle

$-6 < x < -\frac{1}{2}$

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$2x^2 + 13x = -6$
 $+6 +6$

$2x^2 + 13x + 6 = 0$

$(x+12)(x+1) = 0$

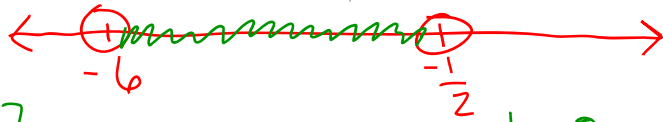
$(x+6)(2x+1) = 0$

$x+6=0$
 $-6=-6$

$x = -6$

$2x+1=0$
 $-1=-1$

$\frac{2x}{2} = \frac{-1}{2}$
 $x = -\frac{1}{2}$



$\frac{-7}{2x^2+13x < -6}$
 $2(-7)^2 + 13(-7) < -6$
 $98 - 91 < -6$
 $7 < -6$
False

$\frac{-1}{2x^2+13x < -6}$
 $2(-1)^2 + 13(-1) < -6$
 $-11 < -6$
True

$\frac{0}{2x^2+13x < -6}$
 $2(0)^2 + 13(0) < -6$
 $0 < -6$
False

$-6 < x < -\frac{1}{2}$

EXAMPLE 4 $3x^2 \geq 13x + 10$

Name: _____

Date: _____

Solving Quadratic Inequalities

Find the solution set for each inequality:

1. $x^2 - x - 12 < 0$

2. $3x^2 + 2x > 1$

3. $x^2 - 16 \geq 0$

4. $5x^2 - 4x - 1 > 0$

5. $x^2 - 2x - 35 \geq 0$

6. $x^2 - 5x + 6 < 0$