

Solving and Graphing Inequalities in One Variable

The Golden Rule of Inequalities
 you must flip your inequality sign
 when you multiply or divide by
 a negative number

1. Solve Inequality
2. Circle the number (constant)
3. Open or closed?
4. Shade to the right or left
5. Check Work!

Open Circle $< , >$
Closed Circle \leq , \geq

Example: Solve and Graph

$$5 - 3x \leq 13 + x$$

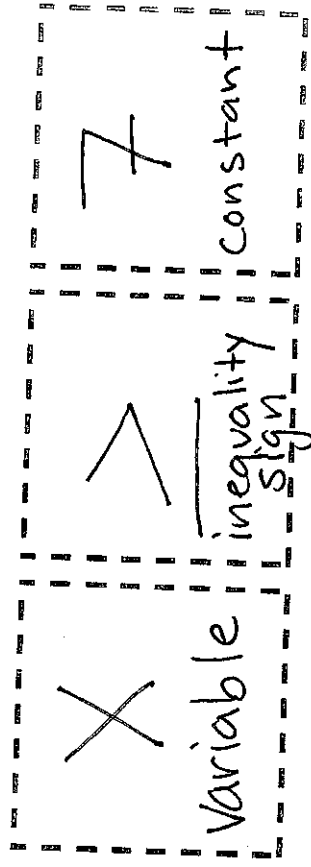
$$-1x \quad | \quad -/x$$

$$\frac{-4x \leq 13}{-4} \quad | \quad \frac{-}{-4}$$

$$\frac{-4x \leq 8}{-4} \quad | \quad \frac{-}{-4}$$

$$x \geq -2 \quad \text{*golden rule*}$$


Inequalities: Order Matters!



Solving Inequalities

- Solving inequalities follows the same procedures as solving equations.
- There are a few special things to consider with inequalities:
 - We need to look carefully at the inequality sign.
 - We also need to graph the solution set.

Review of Inequality Signs

- > greater than
- < less than
- ≥ greater than or equal
- ≤ less than or equal

How to graph the solutions

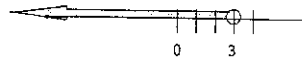
- > Graph any number *greater than* ...
open circle, line to the right $\circ \rightarrow$
- < Graph any number *less than* ...
open circle, line to the left $\leftarrow \circ$
- ≥ Graph any number *greater than or equal to* ...
closed circle, line to the right $\bullet \rightarrow$
- ≤ Graph any number *less than or equal to* ...
closed circle, line to the left $\leftarrow \bullet$

Solve the inequality:

$$x + 4 < 7$$

$$\begin{array}{r} -4 \quad -4 \\ x < 3 \end{array}$$

- Subtract 4 from each side.
- Keep the same inequality sign.
- Graph the solution.
- Open circle, line to the left.



There is one special case.

- Sometimes you may have to *reverse* the direction of the inequality sign!!
- That only happens when you *multiply or divide* both sides of the inequality by a negative number.

Example:

$$\text{Solve: } -3y + 5 > 23$$

$$\begin{array}{r} -5 \quad -5 \\ -3y > 18 \end{array}$$

$$\begin{array}{r} -3y > 18 \\ -3 \quad -3 \end{array}$$

$$y < -6$$

$$y < -6$$

- Subtract 5 from each side.
- Divide each side by negative 3.
- Reverse the inequality sign.
- Graph the solution.
- Open circle, line to the left.



Know:

at least $x \geq \#$

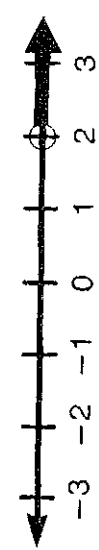



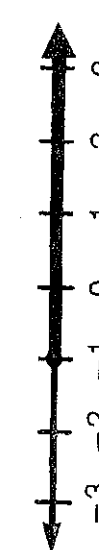

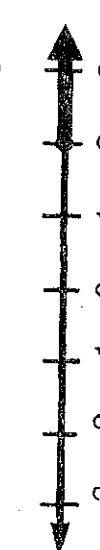
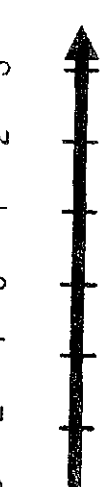

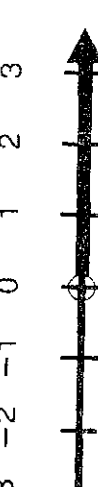
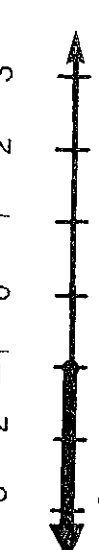
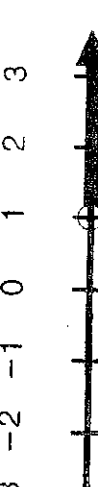
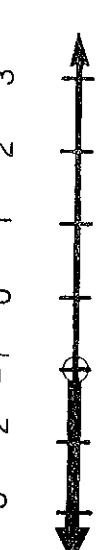




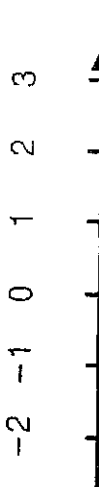
at most $x \leq \#$

no more than $x \leq \#$

no less than $x \geq \#$

What Happened When the Crossword Puzzle Champion Died?

Find the graph of the solution set of each inequality below in the corresponding column of graphs. Notice the letter next to it. Write this letter in each box containing the number of that exercise. Keep working and you will find out about this grave event.

1 $x < 2$		10 $x < 1$	
2 $x \leq 2$		11 $1 < x$ $x > 1$	
3 $x > 2$		12 $-3 \leq x$	
4 $x \geq 2$		13 $x > -3$	
5 $x \neq 1$		14 $x \neq -1$	
6 $x < -1$		15 $0 \geq x$	
7 $x > -1$		16 $0 \leq x$	
8 $x \leq -1$		17 $0 > x$	
9 $x \geq -1$		18 $0 < x$	

1	8	6	11	16	5	14	15	6	17	8	15	10	18	15	7	17	3	2	13	4	13	17	6	15	9	8	1	4	12	14	3	18	18		
T	H	E	Y	B	U	R	I	E	D	H	I	M	S	I	X	D	O	W	N	A	N	D	E	I	J	I	G	H	T	A	C	R	O	S	S

Steps to Solving

Equations vs. Inequalities

1 Undo parentheses (use distributive property)

$$\begin{aligned} -(x+7) &= 8x - 25 \\ -x-7 &= 8x - 25 \end{aligned}$$

2 Combine like terms (same side/ opposite side)

$$\begin{aligned} -x-7 &= 8x-25 \\ -8x-8x & \\ \hline -9x-7 &= -25 \end{aligned}$$

3 Undo Addition and Subtraction

$$\begin{aligned} -9x-7 &= -25 \\ +7 &+7 \\ -9x &= -18 \end{aligned}$$

4 Undo Multiplication and Division

$$\begin{aligned} -9x &= -18 \\ \frac{-9}{-9} &\frac{-18}{-9} \end{aligned}$$

$x=2$ **One Solution**

1 Undo parentheses (use distributive property)

$$\begin{aligned} -(x+7) &> 8x - 25 \\ -x-7 &> 8x - 25 \end{aligned}$$

2 Combine like terms (same side/opposite side)

$$\begin{aligned} -x-7 &> 8x-25 \\ -8x-8x & \\ \hline -9x-7 &> -25 \end{aligned}$$

3 Undo Addition and Subtraction

$$\begin{aligned} -9x-7 &> -25 \\ +7 &+7 \\ -9x &> -18 \end{aligned}$$

4 Undo Multiplication and Division

$$\begin{aligned} -9x &> -18 \\ \frac{-9}{-9} &\frac{-18}{-9} \end{aligned}$$

$x < 2$ **Many Solutions**

Golden Rule!

Note: When you multiply or divide by a negative number, the sense of the inequality changes. For example $<$ becomes $>$ or $>$ become $<$

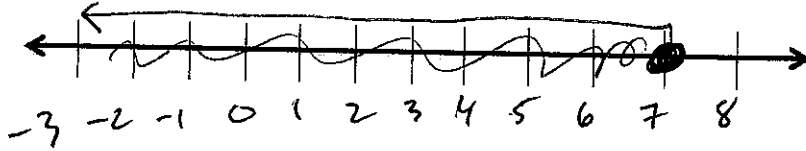
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Inequalities Practice

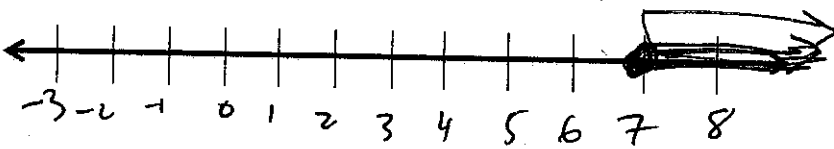
Write, evaluate, and graph the following inequalities:

1. Six increased by twice a number is no more than 20.



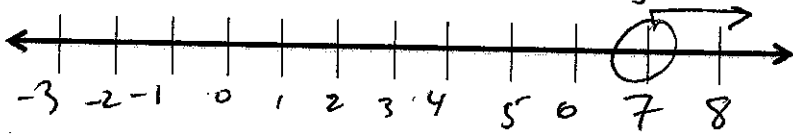
$$\begin{array}{r} 6 + 2x \leq 20 \\ -6 \quad -6 \\ \hline 2x \leq 14 \\ \frac{2x}{2} \leq \frac{14}{2} \\ x \leq 7 \end{array}$$

2. Three times some number, decreased by five is no less than 16.



$$\begin{array}{r} 3x - 5 \geq 16 \\ +5 \quad +5 \\ \hline 3x \geq 21 \\ \frac{3x}{3} \geq \frac{21}{3} \\ x \geq 7 \end{array}$$

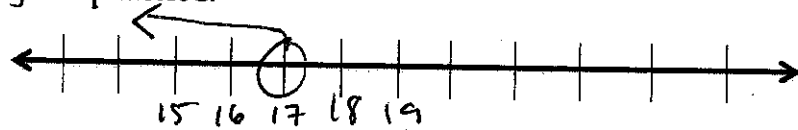
3. 10 is less than some number increased by three.



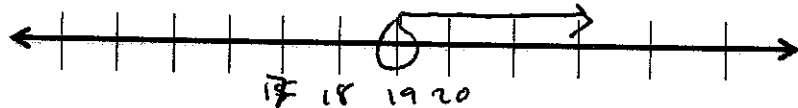
$$\begin{array}{r} 10 < x + 3 \\ -3 \quad -3 \\ \hline 7 < x \rightarrow \text{swap places} \\ x > 7 \end{array}$$

Evaluate and graph the following inequalities:

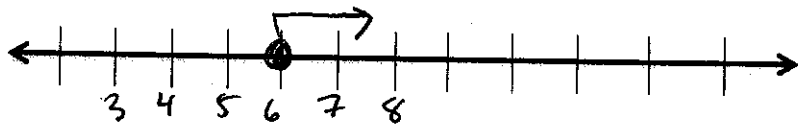
4. $y - 7 < 10$
 $+7 \quad +7$
 $y < 17$



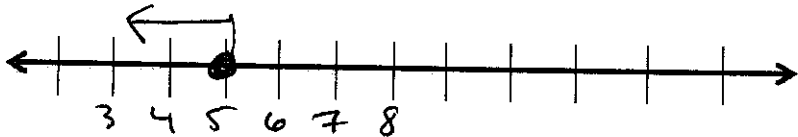
5. $x + 17 > 36$
 $-17 \quad -17$
 $x > 19$



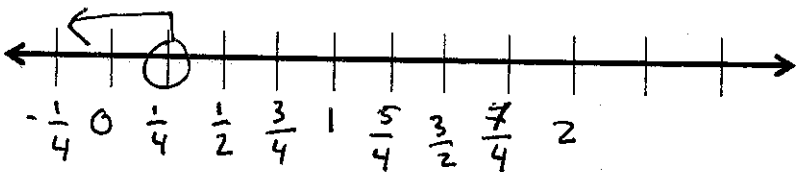
6. $5 + \frac{n}{2} \geq 8$
 $-5 \quad -5$
 $2 \cdot \frac{n}{2} \geq 3 \cdot 2$
 $n \geq 6$



7. $3n - 3 \leq 12$
 $+3 \quad +3$
 $\frac{3n}{3} \leq \frac{15}{3}$
 $n \leq 5$



8. $x + \frac{1}{2} < \frac{3}{4}$
 $-\frac{1}{2} \quad -\frac{1}{2}$
 $x < \frac{1}{4}$



Name: _____

Score: _____

Writing Inequalities

Write the inequality that best describes each graph :

1)

Inequality: $x \geq 44$

2)

Inequality: $x > 18$

3)

Inequality: $x < -2$

4)

Inequality: $x \geq 24$

5)

Inequality: $x \leq -4$

6)

Inequality: $x \leq -15$

7)

Inequality: $x > 30$

8)

Inequality: $x \geq 36$

9)

Inequality: $x < 7$

10)

Inequality: $x \leq -1$

Name: _____

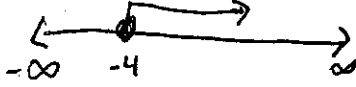
~~pe~~
bracket only
on closed circles!

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Identifying Solutions

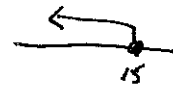
Choose the correct interval that best describes each inequality.

1) $x \geq -4$



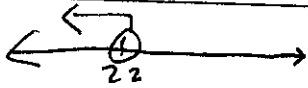
- a) $(-4, \infty)$ b) $(-\infty, -4)$
c) $(-\infty, -4]$ d) $[-4, \infty)$

2) $15 \geq x$ $x \leq 15$



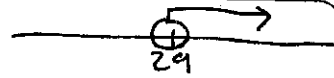
- a) $(-\infty, 15]$ b) $[15, \infty)$
c) $(-\infty, 15)$ d) $(15, \infty)$

3) $x < 22$



- a) $[22, \infty)$ b) $(-\infty, 22)$
c) $(-\infty, 22]$ d) $(22, \infty)$

4) $x > 29$



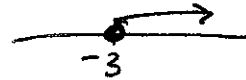
- a) $(29, \infty)$ b) $[29, \infty)$
c) $(-\infty, 29]$ d) $(-\infty, 29)$

5) $18 \leq x$ $x \geq 18$



- a) $(-\infty, 18)$ b) $(18, \infty)$
c) $[18, \infty)$ d) $(-\infty, 18]$

6) $x \geq -3$



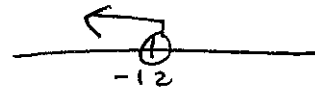
- a) $(-\infty, -3]$ b) $(-3, \infty)$
c) $(-\infty, -3)$ d) $[-3, \infty)$

7) $x > 26$



- a) $(-\infty, 26)$ b) $(26, \infty)$
c) $(-\infty, 26]$ d) $[26, \infty)$

8) $x < -12$



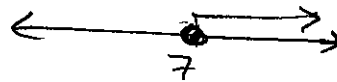
- a) $(-\infty, 12]$ b) $(-\infty, -12]$
c) $(-\infty, -12)$ d) $(-\infty, 12)$

9) $x \leq -21$



- a) $(-\infty, -21]$ b) $(-\infty, -21)$
c) $(-21, \infty)$ d) $[-21, \infty)$

10) $x \geq 7$



- a) $[-7, \infty)$ b) $[7, \infty)$
c) $(-7, \infty)$ d) $(7, \infty)$

Solving & Graphing Inequalities

Solve each inequality and graph the solution, and write in interval notation

1) $-1 > \frac{x+4}{2}$ $x < -6$
 $\frac{-2 > x+4}{-4}$ $(-\infty, -6)$
 $\frac{-6 > x}{-6}$

2) $-2x + 8 \leq 24$ $x \geq -8$
 $\frac{-2x \leq 16}{-2}$ $[-8, \infty)$
 golden rule!

3) $-6x - 13 \geq 17$ $x \leq -5$
 $\frac{-6x \geq 30}{-6}$ $(-\infty, -5]$
 golden rule!

4) $16 < \frac{x}{4} + 9$ $x > 28$
 $\frac{7 < \frac{x}{4}}{4}$ $(28, \infty)$
 $28 < x$

5) $-5x \leq -8x + 21$ $x \leq 7$
 $\frac{3x \leq 21}{3}$ $(-\infty, 7]$

6) $5 \geq -3x - 10$ $x \geq -5$
 $\frac{15 \geq -3x}{-3}$ $[-5, \infty)$
 Golden Rule!

7) $-7x - 19 > 23$ $x < -6$
 $\frac{-7x > 42}{-7}$ $(-\infty, -6)$
 golden rule!

8) $-1 < \frac{x+2}{5}$ $x > -7$
 $\frac{-5 < x+2}{-2}$ $(-7, \infty)$
 $-7 < x$