Analyzing Exponential Functions

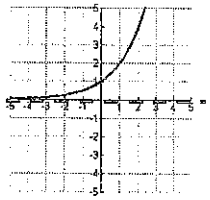
Notes

Characteristics of graphs

Characteristic	How to write the answer
◦ Domain –	(#,#) All reals, x<#, x>#
◦ Range -	(#,#) All reals, y<#, y>#
◦ Zero's – x intercepts	(#,0)
◦ Y intercept –	(0,#)
◦ Rate of change – (slope formula)	# or fraction
◦ Interval of inc/dec –	(#,#)
◦ End Behavior –	
◦ Asymptote –	y = #

Characteristics of Exponential Functions

$y = 2^x$



Domain: $\mathbb{R}$

Range: $y > 0$

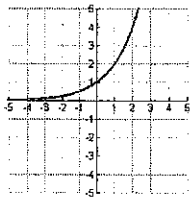
Asymptotes: $y = 0$ Horizontal Asymptote

Zeros: *none*

Intercepts: $(0,1)$

Characteristics of Exponential Functions

$y = 2^x$



Intervals of Increase and Decrease

Increases from $-\infty$ to ∞

Rate of change over the interval $-2 \le x \le 2$

Use the points $(-2, \frac{1}{4})$ and $(2, 4)$ find the rate of change (aka...slope)

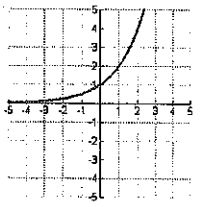
When x = -2: $y = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} = .25$

When x = 2: $y = 2^2 = 4$

$$m = \frac{4 - .25}{2 - (-2)} = \frac{3.75}{4} = .938$$

Characteristics of Exponential Functions

$y = 2^x$



End Behaviors

approaches

$x \to \infty \; f(x) \to \infty$

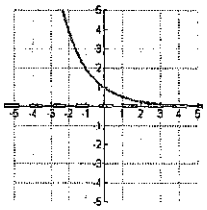
As the value of x approaches positive infinity (∞) the value of the function approaches ∞.

$x \to -\infty \; f(x) \to 0$

As the value of x approaches negative infinity (-∞) the value of the function approaches but NEVER reaches 0.

Characteristics of Exponential Functions

$y = \left(\frac{1}{2}\right)^x$



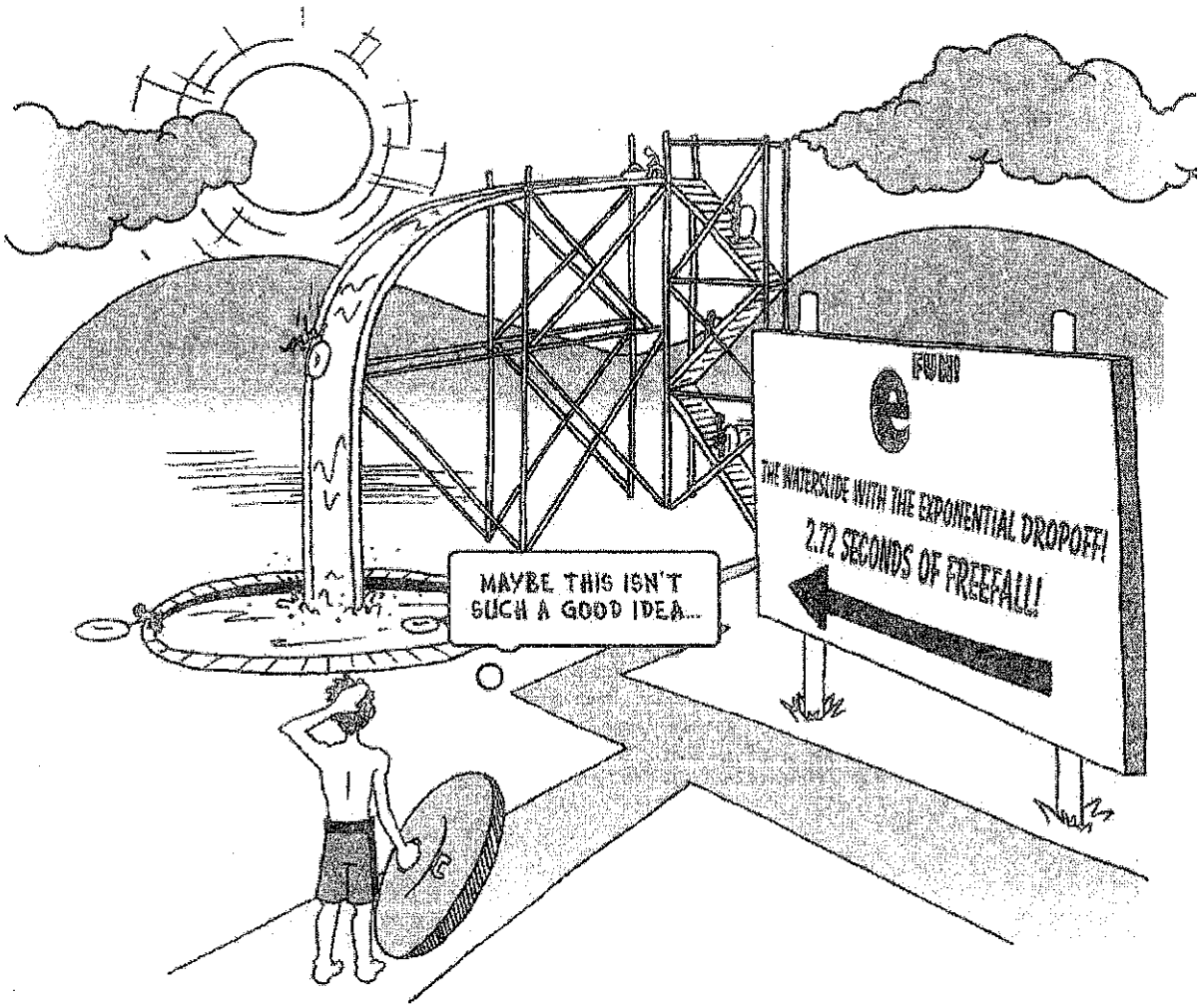
Domain: $\mathbb{R}$

Range: $y > 0$

Asymptotes: $y = 0$ Horizontal Asymptote

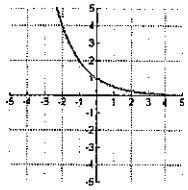
Zeros: *none*

Intercepts: $(0,1)$


FUN!
e
THE WATERSLIDE WITH THE EXPONENTIAL DROPOFF!
2.72 SECONDS OF FREEFALL!
MAYBE THIS ISN'T SUCH A GOOD IDEA...

$y = \left(\frac{1}{2}\right)^x$

Characteristics of Exponential Functions



Intervals of Increase and Decrease

Decreases from $-\infty$ to ∞

Rate of change over the interval $-2 \le x \le 2$

Use the points $(-2, \underline{4})$ and $(2, \underline{\frac{1}{4}})$ find the rate of change (aka...slope)

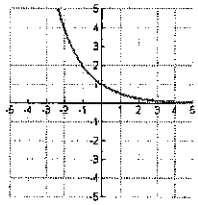
When $x = -2$: $y = \left(\frac{1}{2}\right)^{-2} = \frac{1^{-2}}{2^{-2}} = \frac{2^2}{1^2} = \frac{4}{1} = 4$

When $x = 2$: $y = \left(\frac{1}{2}\right)^{2} = \frac{1^2}{2^2} = \frac{1}{4}$

$$m = \frac{\frac{1}{4} - 4}{2-(-2)} = \frac{\frac{-15}{4}}{4} = -\frac{15}{4} \div 4 = -\frac{15}{4} \cdot \frac{1}{4} = \frac{-15}{16}$$

$y = \left(\frac{1}{2}\right)^x$

Characteristics of Exponential Functions



End Behaviors

approaches

$x \to \infty\ f(x) \to \underline{0}$

As the value of x approaches positive infinity (∞) the value of the function approaches but NEVER reaches 0.

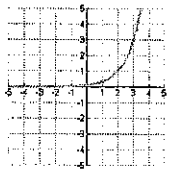
$x \to -\infty\ f(x) \to \underline{\infty}$

As the value of x approaches negative infinity ($-\infty$) the value of the function approaches ∞.

YOUR TURN!!

$y = 3^{x-2}$

Characteristics of Exponential Functions



Domain: All real numbers

Range: y>0

Asymptotes: y=0

Zeros: none

Intercepts: $3^{0-2} = 3^{-2} = \frac{1}{9}$

Intervals of Increase and Decrease:

increase from $-\infty$ to ∞

Rate of change over the interval $-2 \le x \le 2$:

(-2, 1/81) (2, 1) $\frac{1 - \frac{1}{81}}{2-(-2)} = \frac{\frac{80}{81}}{4} = \frac{20}{81}$

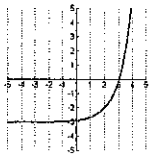
End Behaviors: $x \to \infty\ f(x) \to \infty$

$x \to -\infty\ f(x) \to 0$

YOUR TURN!!

$y = 3^{x-2} - 3$

Characteristics of Exponential Functions



Domain: All real numbers

Range: y>-3

Asymptotes: y=-3

Zeros: (3,0)

Intercepts: $3^{0-2} - 3 = 3^{-2} - 3 = \frac{1}{9} - 3 = \frac{-26}{9}$ (0, -26/9)

Intervals of Increase and Decrease:

increase from $-\infty$ to ∞

Rate of change over the interval $-2 \le x \le 2$:

(-2, -242/81) (2, -2) $\frac{-2 - \frac{-242}{81}}{2-(-2)} = \frac{\frac{80}{81}}{4} = \frac{80}{81} \cdot \frac{1}{4} = \frac{20}{81}$

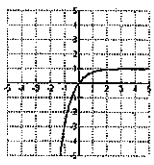
End Behaviors: $x \to \infty\ f(x) \to \infty$

$x \to -\infty\ f(x) \to -3$

YOUR TURN!!

$y = -\left(\frac{1}{4}\right)^x + 1$

Characteristics of Exponential Functions



Domain: All real numbers

Range: Y<1

Asymptotes: y=1

Zeros: (0,0)

Intercepts: $-\left(\frac{1}{4}\right)^0 + 1 = -1 + 1 = 0$ (0,0)

Intervals of Increase and Decrease:

increase from $-\infty$ to ∞

Rate of change over the interval $-2 \le x \le 2$:

(-2, 17) (2, 17/16) $\frac{\frac{17}{16} - 17}{2-(-2)} = \frac{\frac{-255}{16}}{4} = \frac{-255}{16} \cdot \frac{1}{4} = \frac{-255}{64}$

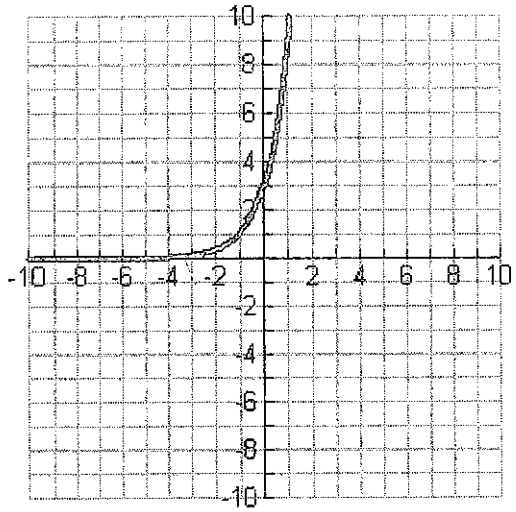
End Behaviors: $x \to \infty\ f(x) \to 1$

$x \to -\infty\ f(x) \to -\infty$

Name: _____ Homework

Analyzing Growth and Decay Exponential Functions

1. $y = 3^{x+1}$



Domain:

Range:

Asymptotes:

Zeros:

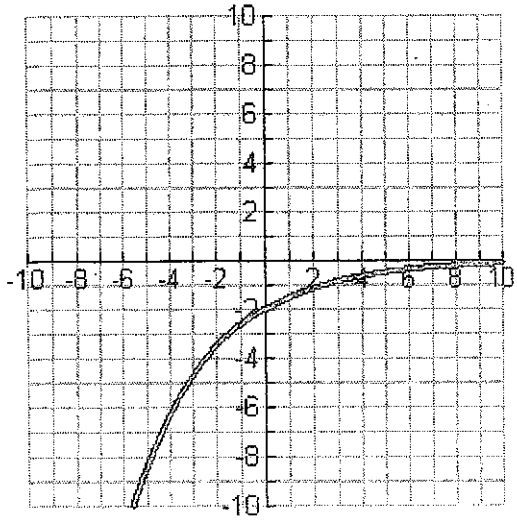
Intercepts:

Intervals of increase and decrease

Rate of change of the interval $-2 \leq x \leq 2$

End behavior

2. $y = -2\left(\frac{3}{4}\right)^x$



Domain

Range:

Asymptotes:

Zeros:

Intercepts:

Intervals of increase and decrease

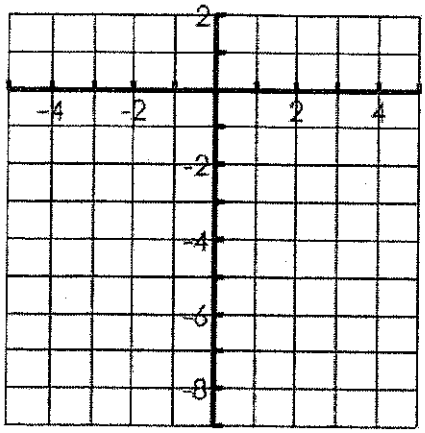
Rate of change of the interval $-2 \leq x \leq 2$

End behavior

Name: _____ Date: _____

Graphing and Characteristics of Exponential Equations

1. $y = -5^x - 3$



Domain_____ Range_____

Asymptote_____

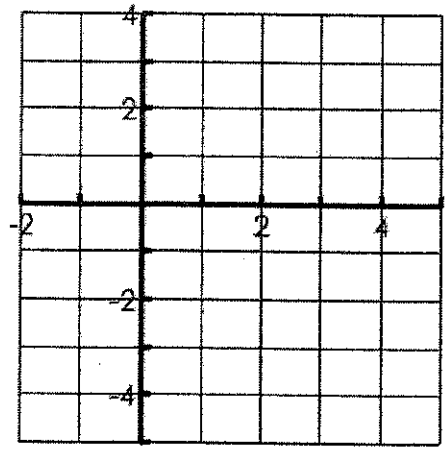
Increasing or Decreasing _____

X-intercept _____ Y-intercept _____

End Behavior $x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____
growth/decay factor_____

2. $y = \left(\frac{1}{3}\right)^{x+3}$



Domain_____ Range_____

Asymptote_____

Increasing or Decreasing _____

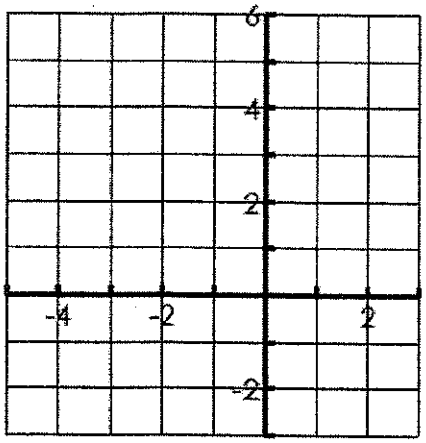
X-intercept _____ Y-intercept _____

End Behavior $x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____

growth/decay factor_____

3. $y = 4^{x-2} - 3$



Domain_____ Range_____

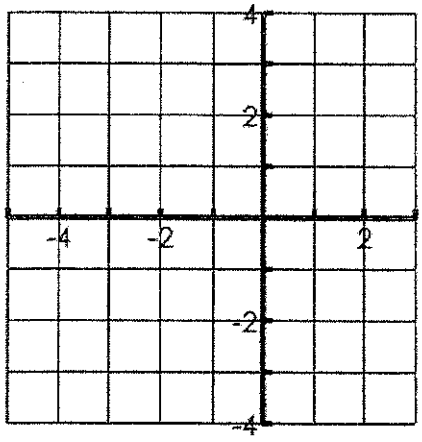
Asymptote_____

Increasing or Decreasing _____

X-intercept _____ Y-intercept _____ End Behavior
$x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____ growth/decay factor_____

4. $y = -2^{x+1} + 1$



Domain_____ Range_____

Asymptote_____

Increasing or Decreasing _____

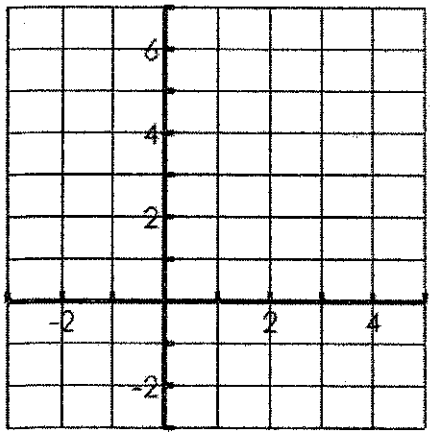
X-intercept _____ Y-intercept _____

End Behavior $x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____

growth/decay factor_____

5. $y = 3^{x-3} + 1$



Domain _____Range_____

Asymptote_____

Increasing or Decreasing _____

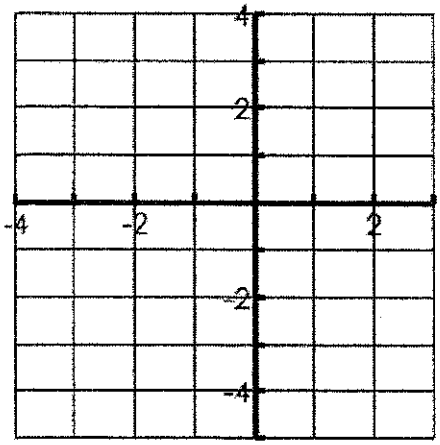
X-intercept _____ Y-intercept _____

End Behavior $x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____

growth/decay factor_____

6. $y = \left(\frac{1}{2}\right)^{x+1} - 2$



Domain_____ Range_____

Asymptote_____

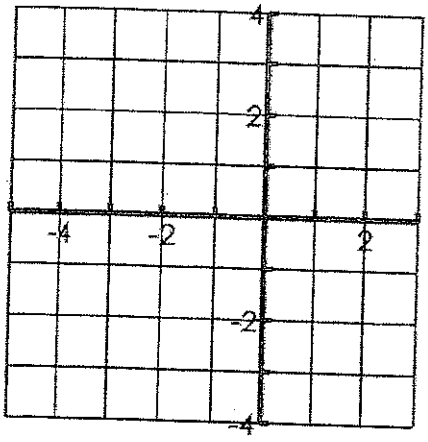
Increasing or Decreasing _____

X-intercept _____ Y-intercept _____

End Behavior $x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____ growth/decay factor_____

4. $y = -2^{x+1} + 1$



Domain_____ Range_____

Asymptote_____

Increasing or Decreasing _____

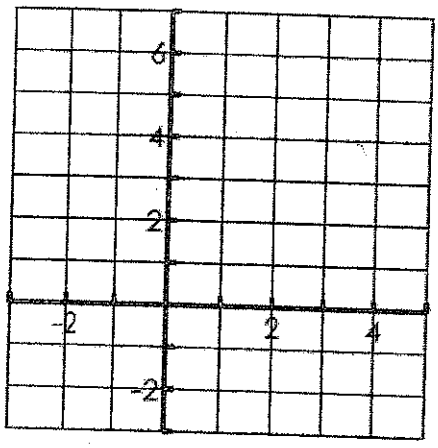
X-intercept _____ Y-intercept _____

End Behavior $x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____

growth/decay factor_____

5. $y = 3^{x-3} + 1$



Domain _____Range_____

Asymptote_____

Increasing or Decreasing _____

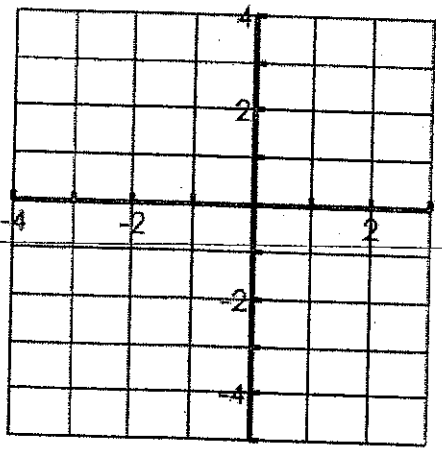
X-intercept _____ Y-intercept _____

End Behavior $x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____

growth/decay factor_____

6. $y = \left(\frac{1}{2}\right)^{x+1} - 2$



Domain_____ Range_____

Asymptote_____

Increasing or Decreasing _____

X-intercept _____ Y-intercept _____

End Behavior $x \to$ _____, $f(x) \to$ _____
$x \to$ _____, $f(x) \to$ _____

Initial Amount: _____ growth/decay factor_____