

(h, k) center $r = \text{radius}$

STANDARD FORM: $(x-h)^2 + (y-k)^2 = r^2$

GENERAL FORM: $ax^2 + by^2 + cx + dy + e = 0$

1. Convert to general form: $x^2 + (y+4)^2 = 9$

① Expand

② Combine Like Terms

③ Set = to 0

Standard

$$\begin{aligned} \text{① } x^2 + (y+4)(y+4) &= 9 \\ \text{② } x^2 + y^2 + 4y + 4y + 16 &= 9 \\ \text{③ } x^2 + y^2 + 8y + 16 - 9 - 9 &= 0 \\ \boxed{x^2 + y^2 + 8y + 7} &= 0 \end{aligned}$$

2. Convert to general form: $(x+1)^2 + (y-4)^2 = 8$

$$\begin{aligned} \text{① } (x+1)(x+1) + (y-4)(y-4) &= 8 \\ x^2 + 1x + 1x + 1 + y^2 - 4y - 4y + 16 &= 8 \\ \text{② } x^2 + 2x + 17 + y^2 - 8y &= 8 \\ x^2 + y^2 + 2x - 8y + 17 - 8 - 8 &= 0 \\ \text{③ } \boxed{x^2 + y^2 + 2x - 8y + 9} &= 0 \end{aligned}$$

CONVERTING FROM GENERAL TO STANDARD FORM!!!

If the quadratic equation isn't in the standard form for a circle, we must first **complete the square** to get it in the correct form.

Steps to complete the square.

First, prepare the terms:

- Group Xs and leave a space.
- Group Ys and leave a space.
- Move the constant and leave 2 blanks.

Then, complete the square:

- ① $\frac{1}{2}$ the linear term and square it. $(\frac{b}{2})^2$
 - ② Add to both sides.
 - ③ Do this for both x and y.
- Factor and simplify.

general

$$\begin{aligned} 1. x^2 + y^2 + 16x - 22y - 20 &= 0 \\ x^2 + 16x + 64 + y^2 - 22y + 121 &= 20 + 64 + 121 \\ (\frac{16}{2})^2 = 8^2 = 64 & \quad (\frac{-22}{2})^2 = (-11)^2 \\ (x+8)(x+8) & \quad (y-11)(y-11) \\ \boxed{(x+8)^2 + (y-11)^2} &= 205 \\ (x-h)^2 + (y-k)^2 = r^2 & \quad \text{center } (-8, 11) \\ 2. x^2 + y^2 - 12x + 8y + 32 &= 0 \quad \text{radius} = \sqrt{205} \\ x^2 - 12x + 36 + y^2 + 8y + 16 &= -32 + 36 + 16 \\ (\frac{12}{2})^2 = 6^2 = 36 & \quad (\frac{8}{2})^2 = 4^2 = 16 \\ \boxed{(x-6)^2 + (y+4)^2} &= 20 \end{aligned}$$

3. $x^2 + y^2 + 2x - 15 = 0$

$$x^2 + 2x + 1 + y^2 = 15 + 1$$

$$\begin{aligned} \frac{2}{2} = 1^2 \\ (x+1)^2 + y^2 &= 16 \end{aligned}$$

C: $(-1, 0)$ R: $\sqrt{16} = 4$

C: $(6, -4)$

R: $\sqrt{20} = 2\sqrt{5}$