Geometry **Name** _____

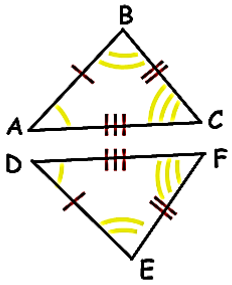
Triangle Congruence: SSS and HL **Date**_____

In similar triangles, we proved that angles are congruent and sides are proportional.
Now we are dealing with congruent triangles.
In congruent triangles, angles are congruent and sides are **congruent.**

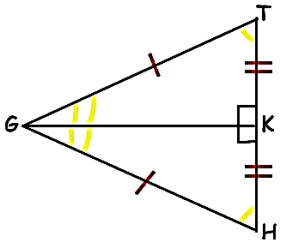
The parts of congruent triangles that "match" are called **corresponding parts.**

Complete the congruence statement for the following.

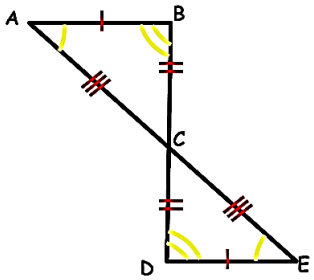
$\Delta ABC \cong \Delta$_____



$\Delta GHK \cong \Delta$_____



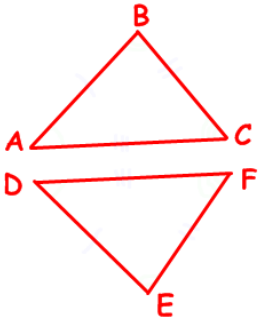
$\Delta ACB \cong \Delta$_____



Example: Given $\Delta ABC \cong \Delta DEF$

Make six congruence statements about the corresponding parts.

Mark the diagrams with hash marks and arcs to identify congruent parts.



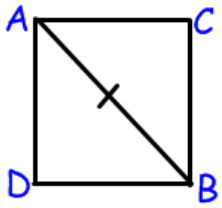
$\angle A \cong$ _____ $\overline{AB} \cong$ _____

$\angle B \cong$ _____ $\overline{AC} \cong$ _____

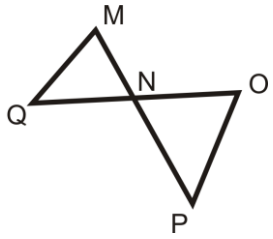
$\angle C \cong$ _____ $\overline{BC} \cong$ _____

There are 4 special properties that allow you to add marks to your triangles:

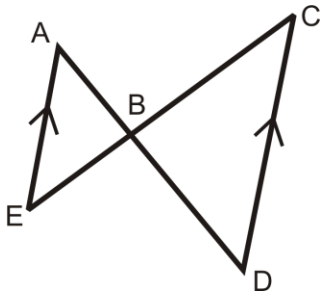
1. Reflexive Property
   a. This is when the two triangles share a side. We state that $\overline{AB} \cong \overline{AB}$



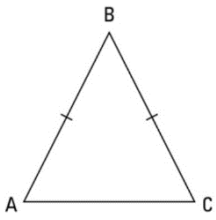
2. Vertical Angles
   a. Vertical angles can be marked congruent in the two triangles. In this case we state (and mark) that $< MNQ \cong < ONP$



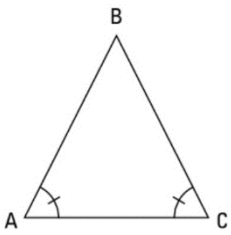
3. Alternate Interior Angles
   a. IF there are parallel lines, then you can mark alternate interior angles. In this case we state (and mark) that $< EAB \cong < BDC$ and also $< AEB \cong < BCD$



4. Base Angle Theorem
   a. In isosceles triangles if two sides are marked congruent then their opposite angles are marked congruent AND if two angles are marked congruent then their opposite sides are marked congruent.
      i. $< BCA \cong < BAC$ since $\overline{AB} \cong \overline{BC}$



      ii. $\overline{AB} \cong \overline{BC}$ since $< BCA \cong < BAC$



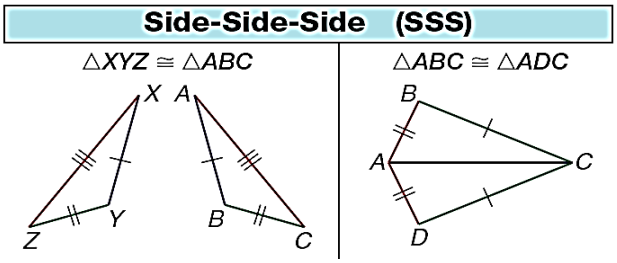
IF IT IS NOT ONE OF THESE FOUR OPTIONS YOU CAN NOT ADD THE MARKS!!!

There are 5 methods to prove that triangles are congruent:

1. Side Side Side (SSS)
2. Hypotenuse Leg (HL)
3. Side Angle Side (SAS)
4. Angle Angle Side (AAS)
5. Angle Side Angle (ASA)

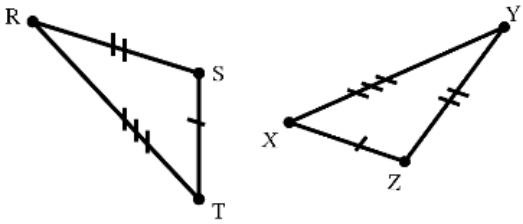
Side Side Side

SSS
If the three sides of one triangle are congruent to the three sides of a second triangle, then the two triangles are congruent.



1. Determine whether the triangles are congruent. If they are, write a congruence statement explaining why they are congruent.

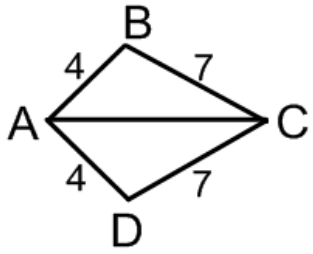
Δ_____ ≅ Δ_____ by _____



YOU TRY! Are these triangles congruent by SSS? If so, name them. Remember your special properties!

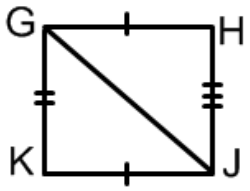
T

1.



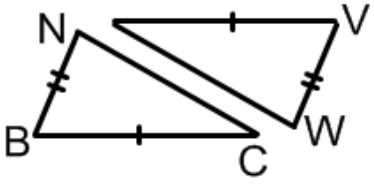
Δ_____≅Δ_____

2.



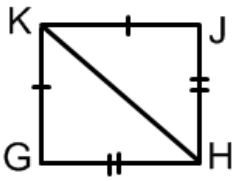
Δ_____≅Δ_____

3.



Δ_____≅Δ_____

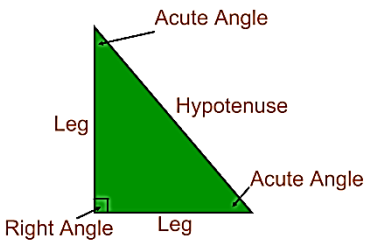
4.



Δ_____≅Δ_____

Hypotenuse Leg

Parts of a Right Triangle

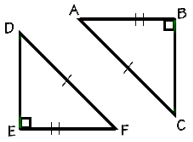


HL

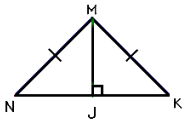
If the hypotenuse and a leg of one right triangle is congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent

Hypotenuse-Leg (HL)

$\Delta DEF \cong \Delta CBA$



$\Delta NMJ \cong \Delta KMJ$

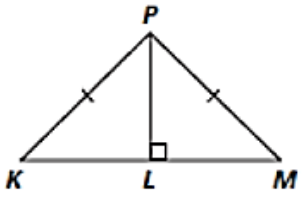


Hypotenuse Leg Checklist:

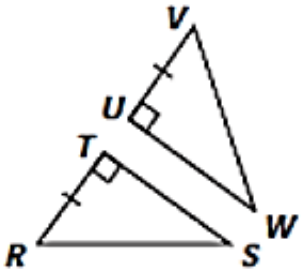
- ✓ Can you add any marks? (ONLY USE THE FOUR!)
- ✓ Are both triangles right triangles? (they must have marked right angles)
- ✓ Are the hypotenuse of both congruent? (draw the arrow to help find this)
- ✓ Is there one pair of congruent legs marked?

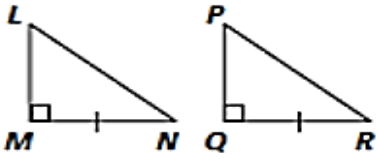
Can you use HL to prove the two triangles congruent?
If yes, write a congruence statement. If not, explain why not.

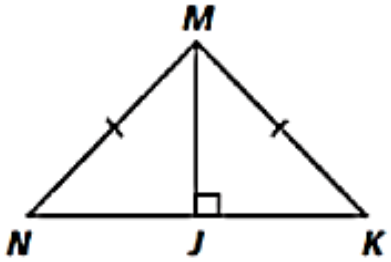
1. _____

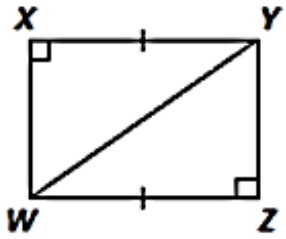


2. _____ 3. _____ 4. _____ 5. _____







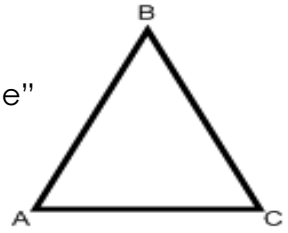


The next three theorems utilize angles:

Included Angle : The angle directly between two sides. "The sandwiched angle"

Name the angle included between $\overline{AB}$ and $\overline{BC}$ _____

$\overline{BC}$ and $\overline{AC}$ _____ $\overline{AC}$ and $\overline{AB}$ _____

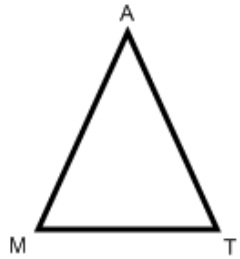


Included Side: The side directly between two angles.

1. In ΔMAT, which side is included between <A and <T? _____

2. In ΔMAT, which side is included between <M and <A? _____

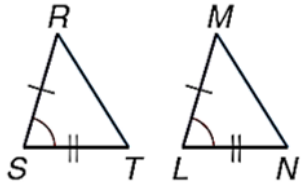
3. Which side is not included between angles A and T? _____



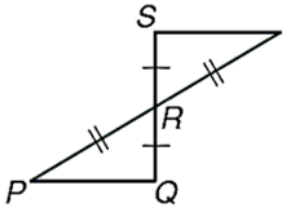
Side Angle Side

Side-Angle-Side (SAS)

$\triangle SRT \cong \triangle LMN$



$\triangle SRT \cong \triangle QRP$



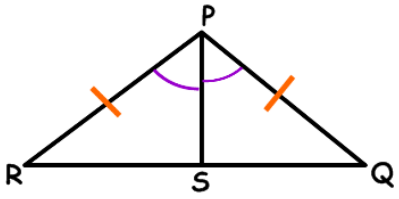
If two sides and the <u>included</u> angle of one triangle are congruent to two sides and the <u>included</u> angle of a second triangle, then the two triangles are congruent.

Side Angle Side Checklist:

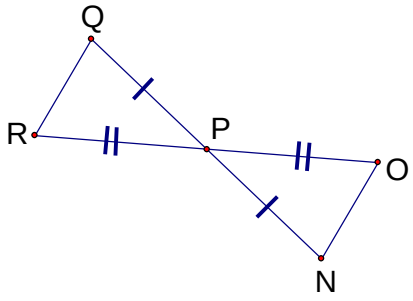
- ✓ Can you add any marks? (Only use the four! Reflexive property, Vertical angles, Alternate Interior Angles, and Base Angle Theorem)
- ✓ Look for two congruent sides with an included angle on both
  - "congruent side then congruent angle then congruent side"

Examples: Write the statement if the triangles are congruent

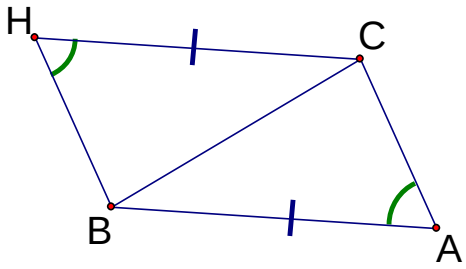
1._____ **by** _____



2. _____ **by** _____



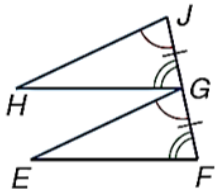
A non-example:
This would not be congruent by SAS! The angles are not included angles



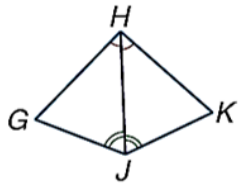
Angle Side Angle

Angle-Side-Angle (ASA)

$\triangle GHJ \cong \triangle FEG$



$\triangle GHJ \cong \triangle KHJ$



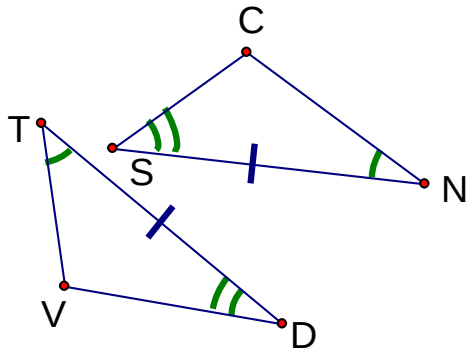
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

Angle Side Angle Checklist:

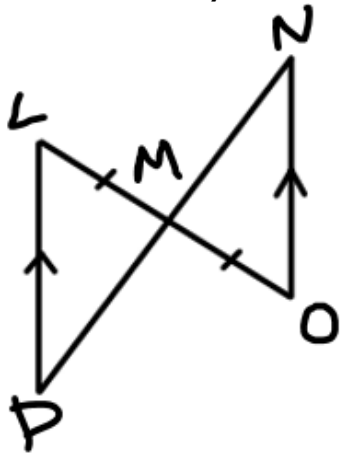
- ✓ Can I add any marks? (ONLY OUR FOUR ALLOWED! Reflexive property, vertical angles, alternate interior angles, or base angle theorem).
- ✓ Look for two congruent angles with an included side
  - "congruent angle then congruent side then congruent angle"

Examples: Write the statement if the triangles are congruent.

1. _____ **by** _____

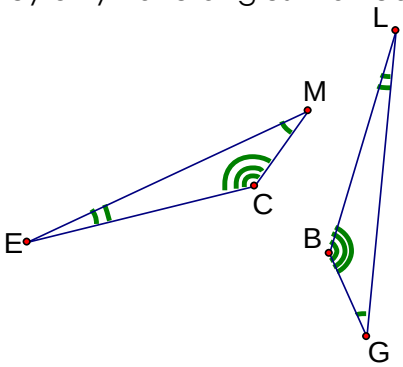


2. _____ **by** _____



Non-Example:

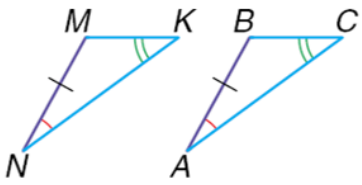
These are not congruent because they only have angles marked (no sides). ANGLE-ANGLE-ANGLE is not a congruency theorem!



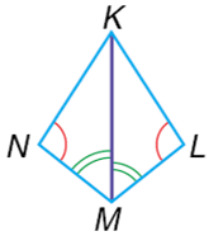
Angle Angle Side

Angle-Angle-Side (AAS)

$\triangle NMK \cong \triangle ABC$



$\triangle NMK \cong \triangle LMK$



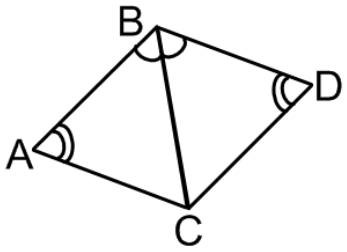
If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded angle of a second triangle, then the two triangles are congruent.

Angle-Angle-Side Checklist:

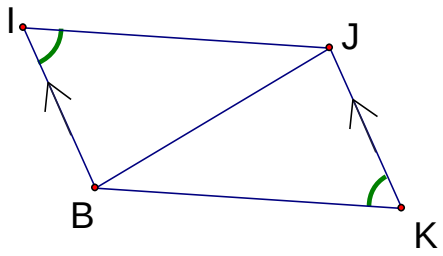
- ✓ Can I add any marks? (ONLY THE FOUR ALLOWED MARKS! Reflexive property, vertical angles, alternate interior angles, base angle theorem)
- ✓ Do I have two congruent angles and a non-included side that is congruent?
  - "congruent angle then congruent angle then congruent side"

Examples: Write the statements if the triangles are congruent

1. _____ **by** _____



2. _____ **by** _____



A non-example:

This would not be an example because there are two congruent pairs of sides and one congruent pairs of angles with the wrong ordering. Here there is side then side then angle, or angle then side then side. There are no bad words in math!